New and Interesting Theorems

Pedro Quaresma[1],

[1] CISUC, Department of Mathematics, University of Coimbra

In *Automated Reasoning: 33 Basic Research Problems*, Larry Wos, write about the problems that computer programs that reason face. Problema 31 it is still open and object of active research:

> What properties can be identified to permit an automated reasoning program to find new and interesting theorems, as opposed to proving conjectured theorems?

Two problems in a single sentence: new and interesting problems. The automatic discovery of new problems is a goal in itself, it has been addressed in specific areas, with different methods. The separation of the "weeds", uninteresting, trivial facts, from the "wheat", new and interesting facts, is much harder, but is being addressed also, by different authors using different approaches.

Paraphrasing, again, Wos, "since a reasoning program can be instructed to draw some (possible large) set of conclusions" what should be the "criteria that permit the program to select from those the ones (if any) that correspond to interesting results."

The different approaches for the automatic discovery of mathematical theorems (and properties) and the proposed metrics to find the interesting ones among all those that were generated, are presented and discussed.

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