

# The existential transversal property: A generalization of homogeneity and its curious behaviour

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Let  $G$  be a permutation group of degree  $n$  on the domain  $\Omega$ , and  $k$  a positive integer with  $k \leq n$ . We say that  $G$  has the  *$k$ -existential transversal property*, or  *$k$ -et*, if there exists a set  $A \subseteq \Omega$  of size  $k$  whose orbit under  $G$  contains transversals for all partitions of  $\Omega$  into  $k$  parts [1]. This property is a substantial weakening of the  *$k$ -universal transversal property* (or  *$k$ -ut*) introduced by Araújo and Cameron [2], which required this condition to hold for all  $k$ -subsets  $A$  of the domain  $\Omega$ .

The original motivation for both of these properties is their relation to the regularity property for transformation monoids of the form  $\langle G, t \rangle$ , where  $t$  is a singular transformation on  $\Omega$ . This is part of a larger research arc, examining how the properties of a transformation monoid are determined by its group of units.

In addition to this application, the  *$k$ -et* property is of interest from the perspective of permutation group theory. Its study uses a large variety of different techniques and is a good example of an application of the classification theorem of finite simple groups. Moreover,  *$k$ -et* turned out to have some surprising properties itself. Most curiously,  *$k$ -et* is not monotone in  $k$  (for  $k \leq n/2$ ), and hence there is no analogue of the Livingstone–Wagner Theorem [3] for  *$k$ -et*.

In this talk we give an introduction to the  *$k$ -et* property and present the nearly complete characterizations of  *$k$ -et* (for  $4 \leq k \leq n/2$ ) from [1].

This is joint work with João Araújo (Universidade Nova) and Peter J. Cameron (St Andrews).

## References

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- [3] LIVINGSTONE, DONALD; WAGNER, ASCHER, *Transitivity of finite permutation groups on unordered sets*, Mathematische Zeitschrift , no. 90, 393 - 403 (1965).