Variational and Topological Methods for Differential Equations

Determining the geometry of the human cornea: a contribution from nonlinear analysis

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In this talk I will present some results, that have been obtained in the last years in [1, 2, 3, 4, 5] and concern the following prescribed anisotropic mean curvature equation with Dirichlet boundary conditions:

$$\begin{cases} -\mathrm{div}\left(\dfrac{\nabla u}{\sqrt{1+|\nabla u|^2}}\right) = -au + \dfrac{b}{\sqrt{1+|\nabla u|^2}} & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega. \end{cases} \tag{1}$$

Here, $\Omega \subset \mathbb{R}^N$ is a bounded regular domain and $a, b > 0$ are real parameters.

This quasilinear problem was introduced in [6, 7] in order to provide a mathematical model for describing the geometry of the human cornea.

I aim to show how various techniques of nonlinear analysis, from elementary to more sophisticated, can successfully be combined to derive a rather complete description of the solvability patterns of problem (1), including existence, uniqueness, regularity, boundary behaviour, stability of solutions, as well as information on the structure of the solution set.

References

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This is a submission for a contributed session