Abstract:
We consider PDE-based Group Convolutional Neural Networks (PDE-G-CNNs) that generalize Group equivariant Convolutional Neural Networks (G-CNNs). In PDE-G-CNNs a network layer is a set of PDE-solvers where geometrically meaningful PDE-coefficients become trainable weights. The underlying PDEs are morphological and linear scale space PDEs on the homogeneous space $M=SE(d)/H$ of positions and orientations, where $SE(d)$ is the roto-translation group and H the subgroup stabilizing a reference position and orientation. The PDEs provide a geometrical and probabilistic PDE-design of the roto-translation equivariant network.
The network is implemented by morphological convolutions with approximations to kernels solving nonlinear HJB-PDEs (for morphological α-scale spaces), and to linear convolutions solving linear PDEs (for linear α-scale spaces). In the morphological setting, the parameter α regulates soft max-pooling over Riemannian balls, whereas in the linear setting the cases $\alpha=1/2$ and $\alpha=1$ correspond to the Poisson and Gaussian semigroup respectively. We prove that our practical analytic approximation kernels are accurate by comparing them to my recent formulas for the various exact probability kernels on M for $d=2,3$, published in QAM-AMS 2008, 2010 ($d=2$), and ADG 2018, Entropy 2019 ($d=3$) in response to an open problem by Mumford.
We build on a key isomorphism between the linear and nonlinear PDEs via the (approximate) Fourier-Cramér transform. This connects the underlying α-stable Lévy processes to Bellman processes on M. The equivariant PDE-G-CNN network implementation consists solely of linear and morphological convolutions on M. Common mystifying nonlinearities in CNNs are now obsolete and excluded.
We present blood vessel segmentation experiments in medical images that show clear benefits of PDE-G-CNNs compared to state-of-the-art G-CNNs: increase of performance along with a *huge reduction* in network parameters. We also present experiments that reveal clear benefits of the equivariant CNNs over normal CNNs on mitosis/cancer detection in medical images.

(this is joint work with colleagues B.M.N. Smets, E.J. Bekkers, J.M. Portegies and M. Veta)